OCR
Oxford Cambridge and RSA

Friday 23 June 2017 – Morning

A2 GCE MATHEMATICS (MEI)

4769/01 Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4769/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Estimation

1 The random variable X has the probability density function

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0,$$

where λ is a positive parameter. $X_1, X_2, \ldots, X_n$ are independent random variables distributed as X.

(i) Show that $\tilde{\lambda}$, the maximum likelihood estimator for λ, is $(\bar{X})^{-1}$ where $\bar{X} = \dfrac{X_1 + X_2 + \ldots + X_n}{n}$.

(You should justify that this gives the **maximum** likelihood.) **[10]**

You are given that

$$E(\tilde{\lambda}) = \frac{n}{n-1}\lambda$$

for $n > 1$ and

$$\text{Var}(\tilde{\lambda}) \approx -\left(\frac{d^2(\ln L)}{d\lambda^2}\right)^{-1}$$

when n is large and where L is the likelihood used in part **(i)**.

(ii) Show that the mean square error in $\tilde{\lambda}$ is approximately

$$\lambda^2\left(\frac{1}{n} + \frac{1}{(n-1)^2}\right),$$

when n is large. **[5]**

(iii) Find the value of k for which $\hat{\lambda} = k(\bar{X})^{-1}$ is an unbiased estimator of λ. **[3]**

(iv) Assuming n is large, show that $\hat{\lambda}$ is a more efficient estimator than $\tilde{\lambda}$. **[6]**

Option 2: Generating Functions

2 The random variable X has a Poisson distribution with parameter λ.

(i) Show that the probability generating function of X is

$$G_X(t) = e^{\lambda(t-1)}.$$ **[4]**

The random variable Y is the sum of n independent random variables each distributed as X.

(ii) Find $G_Y(t)$, the probability generating function of Y, and hence state the distribution of Y, including its parameter(s). **[3]**

(iii) Explain why

$$G_Y(-1) = P(Y \text{ is even}) - P(Y \text{ is odd})$$

and hence find the probability that Y takes an even value. **[6]**

(iv) A discrete random variable has probability generating function $G(t)$ and moment generating function $M(t)$. Show that $M(t) = G(e^t)$. **[2]**

(v) By considering the moment generating function of Y, show that $Z = \dfrac{Y - \lambda n}{\sqrt{\lambda n}}$ has approximately the Normal distribution $N(0, 1)$ when λn is large. **[9]**

Option 3: Inference

3 Two randomly selected groups of people were asked by a psychologist to complete a memory test. Group A took the test using distinctive smells which the psychologist thinks will improve recall. Group B took the test without any distinctive smells being present. The results for the two groups (using arbitrary units) are given below. Higher numbers indicate better recall.

Group A scores:	25.8	27.4	26.2	23.5	28.3	26.4	27.2
Group B scores:	25.6	24.9	23.7	25.9	26.9		

(i) Use a t test at the 5% significance level to investigate whether the presence of distinctive smells improves performance in memory tests. **[10]**

(ii) State two distributional assumptions required in using the t test for these data. **[2]**

(iii) If the assumptions in part **(ii)** are not justified, a non-parametric test could be used. Carry out this alternative test at the 5% significance level to investigate whether the presence of distinctive smells improves performance in memory tests. **[11]**

(iv) If the assumptions in part **(ii)** are justified, why is it preferable to use a t test rather than a non-parametric test? **[1]**

Option 4: Design and Analysis of Experiments

4 **(i)** What is meant by randomised block design? Describe an experimental situation in which it would be appropriate to use randomised block design. Justify why randomised block design is appropriate in this situation. **[5]**

(ii) The usual one-way analysis of variance model can be written as

$$x_{ij} = \mu + \alpha_i + e_{ij}\,.$$

Interpret the terms in the model. State any distributional assumptions required in the e_{ij} term. **[5]**

Two drugs and a placebo are used in a trial on 15 patients, allocated randomly to three different groups. The level of a hormone in the blood, x (in arbitrary units), is measured two hours after the treatment is administered. The results are shown in the table below.

	Placebo	Drug A	Drug B
Hormone level	101	102	94
	105	94	90
	98	86	108
	88	108	100
	87	110	106
$\sum x$	479	500	498
$\sum x^2$	46 143	50 400	49 836

(iii) Conduct the usual one-way analysis of variance test at the 5% significance level to investigate whether there is a difference between hormone levels in the three situations. State your hypotheses carefully and report briefly your conclusions. **[12]**

(iv) Jamie thinks that the analysis of variance test used in part **(iii)** may not be valid. He uses a goodness of fit test at the 5% significance level to investigate whether a Normal distribution can be fitted to the 15 data items. He concludes that a Normal distribution does not fit and he claims that the analysis of variance is therefore not appropriate.

Explain why, although Jamie has carried out a goodness of fit test correctly, his claim about analysis of variance being inappropriate is not correct. **[2]**

END OF QUESTION PAPER

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)	

1 (ii)	
1 (iii)	

1 (iv)

2 (i)	
2 (ii)	

2 (iii)

2 (iv)

2 (v)

3 (i)	
3 (ii)	

3 (iii)	
3 (iv)	

4 (i)	
4 (ii)	

4 (iii)

4 (iv)	

Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4769**: Statistics 4

Advanced GCE

Mark Scheme for June 2017

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the

establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep *’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$L = \lambda e^{-\lambda X_1} \times \lambda e^{-\lambda X_n} \ldots$	M1	Product	
			$= \lambda^n e^{-\lambda(X_1+X_2\ldots)}$	A1		
			$\ln L = n\ln\lambda - \lambda(X_1 + X_2 \ldots)$	M1	Taking logs	
				A1	Correct result Ft their L	
			$\frac{\mathrm{d}(\ln L)}{\mathrm{d}\lambda}$	M1	Differentiating wrt λ	
			$= \frac{n}{\lambda} - (X_1 + X_2 \ldots))$	A1	Ft their L	
			$= 0$	M1	Set to zero and solve No ft	
			$\tilde{\lambda} = \frac{n}{\Sigma X_i}$	A1		
			$= \frac{1}{\bar{X}}$		AG	
			$\frac{\mathrm{d}^2(\ln L)}{\mathrm{d}\lambda^2} = -\frac{n}{\lambda^2}$	M1	Checking for maximum.	
			< 0 therefore maximum.	A1 [10]	Consistent	
	(ii)		$\mathrm{Var}(\tilde{\lambda}) \approx -\left(-\frac{n}{\lambda^2}\right)^{-1}$	M1	Ft from their answer to (i)	
			$= \frac{\lambda^2}{n}$	A1		
			$MSE = \mathrm{Var}(\tilde{\lambda}) + \left(\mathrm{Bias}(\tilde{\lambda})\right)^2$	M1		
			$= \frac{\lambda^2}{n} + \left(\frac{n}{n-1}\lambda - \lambda\right)^2$	A1	Bias correct (before squaring)	
			$= \frac{\lambda^2}{n} + \lambda^2\left(\frac{n}{n-1} - \frac{n-1}{n-1}\right)^2$	A1	Some algebraic manipulation to combine fractions must be seen. AG	
			$= \lambda^2\left(\frac{1}{n} + \frac{1}{(n-1)^2}\right)$	[5]		

Question		Answer	Marks	Guidance
	(iii)	Using $\mathrm{E}(k\bar{X}^{-1}) = k\frac{n}{n-1}\lambda$ Requiring $E(\hat{\lambda}) = \lambda$ $k = \dfrac{n-1}{n}$	M1 M1 A1 [3]	A0 if equation connects $\tilde{\lambda}$ not λ
	(iv)	$\mathrm{MSE}(\hat{\lambda}) = \dfrac{(n-1)^2}{n^2}\mathrm{Var}(\tilde{\lambda})$	M1 M1 A1	Trying to find MSE for $\hat{\lambda}$ Using expectation algebra (possibly incorrectly) Condone $\mathrm{Var}(\tilde{\lambda}) = \frac{\lambda^2}{n}$
		METHOD 1 – using differences		
		$\mathrm{MSE}(\tilde{\lambda}) - \mathrm{MSE}(\hat{\lambda}) = \lambda^2\left(\dfrac{1}{n-1}\right)^2 + \dfrac{2n-1}{n^2}\mathrm{Var}(\tilde{\lambda})$	M1A1	M1 for trying differences, A1 for getting a form which allows final argument.
		Which is positive (for $n > 1$) so $\hat{\lambda}$ is more efficient.	E1	
		METHOD 2 – using ratios		
		$\dfrac{\mathrm{MSE}(\tilde{\lambda})}{\mathrm{MSE}(\hat{\lambda})} = \dfrac{\mathrm{Var}(\tilde{\lambda}) + \lambda^2\left(\frac{1}{n-1}\right)^2}{\frac{(n-1)^2}{n^2}\mathrm{Var}(\tilde{\lambda})}$	M1	M1 for trying ratio, A1 for getting a form which allows final argument.
		$= \dfrac{n^2}{(n-1)^2} + \dfrac{\lambda^2 n^2}{\mathrm{Var}(\tilde{\lambda})(n-1)^4}$ $= \left(\dfrac{n^2}{(n-1)^2}\right)\left(1 + \dfrac{\lambda^2}{\mathrm{Var}(\tilde{\lambda})(n-1)^2}\right)$ $= \left(1 + \dfrac{1}{n-1}\right)^2\left(1 + \dfrac{\lambda^2}{\mathrm{Var}(\tilde{\lambda})(n-1)^2}\right)$	A1	
		Which is a product of terms greater than one (for n>1) so is greater than one so $\hat{\lambda}$ is more efficient.	E1	

Question			Answer	Marks	Guidance	
			METHOD 3 – comparison to $\mathrm{Var}(\tilde{\lambda})$. $\mathrm{MSE}(\hat{\lambda}) = \frac{(n-1)^2}{n^2}\mathrm{Var}(\tilde{\lambda}) < \mathrm{Var}(\tilde{\lambda})$ $\mathrm{MSE}(\tilde{\lambda}) = \mathrm{Var}(\tilde{\lambda}) + \lambda^2\left(\frac{1}{n-1}\right)^2 > \mathrm{Var}(\tilde{\lambda})$ So $\mathrm{MSE}(\tilde{\lambda}) > \mathrm{MSE}(\hat{\lambda})$ so $\hat{\lambda}$ is more efficient	M1 A1 E1 [6]	Attempting to compare to $\mathrm{Var}(\tilde{\lambda})$ Two correct comparisons	

2	(i)		$P(X=r)=\frac{e^{-\lambda}(\lambda^r)}{r!}$ $G_X(t)=E(t^X)=\frac{e^{-\lambda}}{0!}+\frac{e^{-\lambda}}{1!}\lambda t+\frac{e^{-\lambda}}{2!}\lambda^2t^2\ldots$ $\left(=e^{-\lambda}\left(1+(\lambda t)+\frac{(\lambda t^2)}{2!}\ldots\right)\right)$ $=e^{-\lambda}(e^{\lambda t})$ $=e^{\lambda(t-1)}$	B1 M1 A1 A1 AG [4]		
	(ii)		$G_Y(t)=\left(e^{\lambda(t-1)}\right)^n$ $=e^{\lambda n(t-1)}$ Poisson with parameter or mean λn	M1 A1* A1 [3]	 Dependent on *	
	(iii)		$G_Y(-1)=P(Y=0)-P(Y=1)+P(Y=2)-P(Y=3)\ldots$ $=P(Y=\text{even})-P(Y=\text{odd})$ $G(1)=P(Y=\text{Even})+P(Y=\text{odd})$ $=1$ $G(-1)=e^{-2\lambda n}$ $P(Y=\text{even})=\frac{1}{2}(G(1)+G(-1))$ $=\frac{1+e^{-2\lambda n}}{2}$	B1 AG M1 A1 B1 M1 A1 [6]	 Considering G(1) Some attempt at linking G(1) and G(-1).	
	(iv)		$M(t)=E(e^{tX})$ $=E((e^t)^X)$ $=G(e^t)$	B1 B1 AG [2]		

	(v)		$M_Y(t) = e^{\lambda n(e^t - 1)}$	B1		
			$M_Z(t) = e^{-\sqrt{\lambda n}t} e^{\lambda n\left(e^{\frac{t}{\sqrt{\lambda n}}} - 1\right)}$	M1 A1 A1	Attempting to use linear transformation. $e^{-\sqrt{\lambda n}t}$ $e^{\frac{t}{\sqrt{\lambda n}}}$	
			EITHER $= e^{-\sqrt{\lambda n}t} e^{\lambda n\left[1 + \frac{t}{\sqrt{\lambda n}} + \frac{t^2}{2\lambda n} + \frac{t^3}{6(\lambda n)^{1.5}} \ldots - 1\right]}$	M1 A1	Attempting to expand the exponent. Correct expansion.	
			$= e^{\frac{t^2}{2} + \frac{t^3}{\sqrt{\lambda n}} \ldots}$	A1*	Correct simplification.	
			OR $\ln M_Z(t) = -\sqrt{\lambda n}t + \lambda n\left(e^{\frac{t}{\sqrt{\lambda n}}} - 1\right)$			
			$= -\sqrt{\lambda n}t + \lambda n\left(1 + \frac{t}{\sqrt{\lambda n}} + \frac{t^2}{2\lambda n} + \frac{t^3}{6(\lambda n)^{1.5}} \ldots - 1\right)$	M1 A1	Attempting to expand the exponent. Correct expansion.	
			$= \frac{t^2}{2} + \frac{t^3}{\sqrt{\lambda n}} \ldots$	A1*	Correct simplification.	
			THEN $M_Z(t) \approx e^{\frac{t^2}{2}}$ since later terms negligible for large λn.	E1	Dep A1* Some explanation required.	
			Which is the moment generating function of the N(0,1) distribution.	E1	Allow for knowledge of mgf for N(0,1)	
				[9]		

3	(i)		$H_o: \mu_A = \mu_B$ $H_o: \mu_A > \mu_B$	B1 B1	Both. Appropriate one-tailed test. Using μ (or stating population in words)	
			$s_p = 1.40(4991)$ or $s_p^2 = 1.97(40)$	M1A1		
			$t = \dfrac{(26.4 - 25.4)}{1.40\sqrt{\frac{1}{7}+\frac{1}{5}}}$	B1 M1	Numerator Denominator	
			≈ 1.22	A1	Ft their s_p^2	
			$\nu = 10$ $p = 0.126$	M1 A1		
			or critical value: t =1.812 Not significant or accept H_o		Or $p = 0.126 > 0.05$ or $t = 1.22 < 1.812$	
			Not enough evidence to say that smell improves memory scores, on average.	A1 [10]	Consistent with their working, without overclaiming.	
	(ii)		Normality of both populations Populations have equal variances	B1 B1 [2]		

	(iii)		Choosing Wilcoxon rank sum test or Mann Whitney. H_0: Difference of medians in populations of group A and Group B is zero H_1: Difference of medians between population of Group A and that of Group B is positive Ranking A Rank B Rank 25.8 5 25.6 4 27.4 11 24.9 3 26.2 7 23.7 2 23.5 1 25.9 6 28.3 12 26.9 9 26.4 8 27.2 10 $W = 4 + 3 + 2 + 6 + 9 = 24$ or $T = 24 - \frac{1}{2} \times 5 \times 6 = 9$ Refer to $W_{5,7}$ or $MW_{5,7}$	B1 B1 B1 M1 A1 M1 A1 M1	Accept "population median A=Population median B" for B1 Must refer to population Accept "median A > median B" ie One tailed test Must be of both groups, but possibly in reverse order. Ignore issues with tied ranks. Choosing correct group to sum.	
			So lower critical value is 21 (or 6) not significant Therefore no significant evidence that smell contributes to memory.	A1 M1 A1 [11]	Use of table, correct critical value chosen Ft their statistic and critical value In context, without overclaiming	
	(iv)		The t-test is more powerful, or lower probability of type II error.	E1 [1]	Provided no unsuitable reasons given	

4	(i)		RBD is the arrangement of experimental units into blocks (or groups) where each block takes a different value (or label) of a nuisance factor. For their description: Clear description of an appropriate situation With response and control variables identified (allow all appropriate terminology) Nuisance factor / blocking clearly identified. Justification referencing why this factor should be eliminated from analysis	B1 B1 B1 B1 B1 [5]		
4	(ii)		μ = **population** grand mean for whole experiment α_i = (population) mean by which i^{th} "treatment" or "group" differs from μ e_{ij} are "errors" or "background variation" or "residuals" distributed normally **and** independently (with mean 0) and with the same variance in each "treatment"	B1 B1 B1 B1 B1 [5]		

	(iii)		$H_0: \mu_p = \mu_A = \mu_B$ H_1: Means not all equal.	B1	Both hypotheses correct. Must use population mean (in either symbols or words). Not $H_1: \mu_p \neq \mu_A \neq \mu_B$ Can be in terms of α_i	
			$Total\ SS = 46143 + 50400 + 49836 - (479 + 500 + 498)^2/15$ $= 146379 - \frac{1477^2}{15}$	M1		
			$= 943.733$	A1	art 944	
			$SS_G = \frac{46143}{5} + \frac{50400}{5} + \frac{49836}{5} - \frac{1477^2}{15}$	M1		
			$= 53.7333$	A1	art 54	
			$SS_E = SS_T - SS_G$ $= 890$	M1		
			$df_G = 2$ $df_E = 12$	B1	For both degrees of freedom correct	
			$MS_G = \frac{53.7333}{2} = 26.8666$ $MS_E = \frac{890}{12} = 74.1666$	M1	For either MS_G or MS_E	
			$F = \frac{MS_G}{MS_E} = 0.362$	M1A1	art 0.36	
			$p - value = 0.703$ or $F_{crit} = 3.89$	M1		
			No significant evidence of difference between the (population mean of the) hormone levels in the 3 groups.	A1 [12]		

	(iv)		Any two points from: Assumptions only require residuals / error term to be normal. Each subgroup should be tested for normality separately. The superposition of normal distributions will not necessarily be normal.	E1E1 [2]	Allow sensible alternatives.	

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	49	45	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	55	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	54	49	45	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	67	61	55	49	43	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	57	52	47	42	38	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	56	50	44	37	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	57	49	41	34	27	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	58	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	63	57	51	46	41	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	52	46	41	36	31	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	53	48	43	39	35	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	58	53	48	43	37	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	48	41	34	27	0

		UMS	100	80	70	60	50	40	0
4798	01 FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
		UMS	100	80	70	60	50	40	0

GCE Statistics (MEI)

			Max Mark	a	b	c	d	e	u
G241	01 Statistics 1 MEI (Z1)	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G242	01 Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
		UMS	100	80	70	60	50	40	0
G243	01 Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
		UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

			Max Mark	a	b	c	d	e	u
G244	01 Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02 Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
		UMS	100	80	70	60	50	40	0
G245	01 Statistics 1 MEI	Raw	72	61	55	49	43	37	0
		UMS	100	80	70	60	50	40	0
G246	01 Decision 1 MEI	Raw	72	52	46	41	36	31	0
		UMS	100	80	70	60	50	40	0

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Level 3 Certificate and FSMQ raw mark grade boundaries June 2017 series

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Level 3 Certificate Mathematics for Engineering

				Max Mark	a*	a	b	c	d	e	u
H860	01	Mathematics for Engineering		This unit has no entries in June 2017							
H860	02	Mathematics for Engineering									

Level 3 Certificate Mathematical Techniques and Applications for Engineers

				Max Mark	a*	a	b	c	d	e	u
H865	01	Component 1	Raw	60	48	42	36	30	24	18	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H866	02	Critical maths	Raw	60*	48	42	36	30	24	0
		*Component 02 is weighted to give marks out of 72	Overall	144	112	97	83	70	57	0

Level 3 Certificate Mathematics - Quantitive Problem Solving (MEI) (GQ Reform)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	54	47	40	34	28	0
H867	02	Statistical problem solving	Raw	60*	41	36	31	27	23	0
		*Component 02 is weighted to give marks out of 72	Overall	144	103	90	77	66	56	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	72	63	55	47	39	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0